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EFFECTFUL STREAMS FOR DATAFLOW PROGRAMMING

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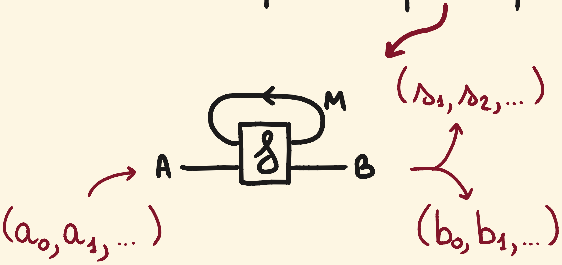
MOTIVATION

Semantics of dataflow programs with effects.



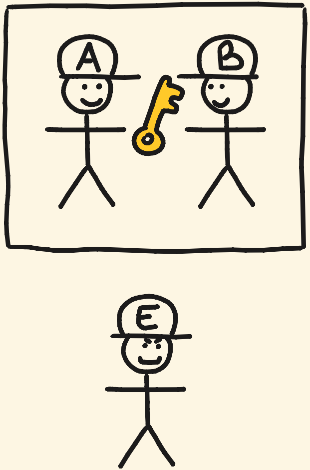
probability
non-determinism
shared memory
⋮

[EDL, Giovanni de Felice and Mario Román
Monoidal streams for dataflow programming (2022) LICS]

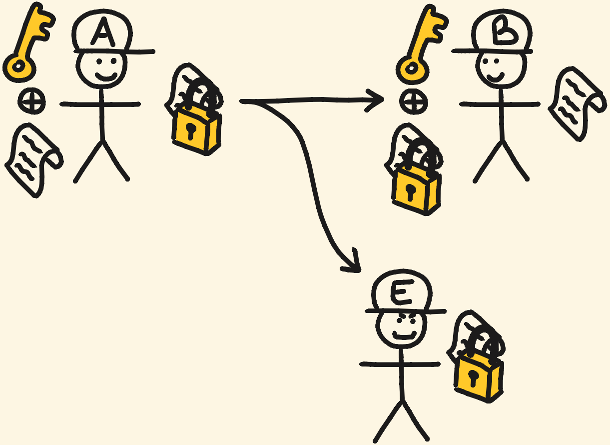
[Filippo Bonchi, EDL, Mario Román
Effectful Mealy machines: bisimulation and trace (2024) preprint]

ONE-TIME PAD PROTOCOL

1. share a key through a secure channel



2. send an encrypted message through a public channel



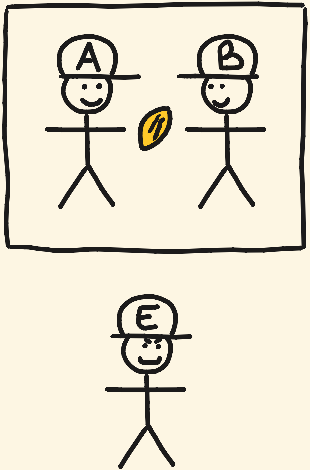
REPEATING THE ONE-TIME PAD

Sending n messages securely requires n private keys
↳ not very useful

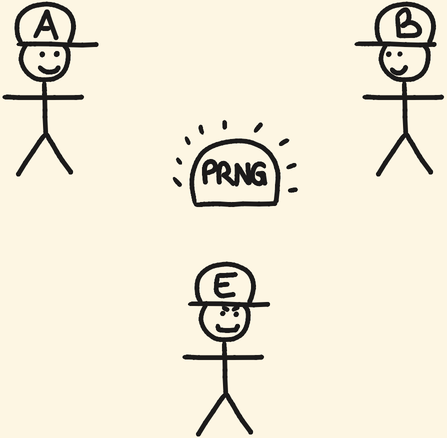
⇒ - privately share a seed
- use identical pseudorandom number generator to obtain a new key for each message

STREAM CIPHER PROTOCOL (1)

1. share a seed through a secure channel

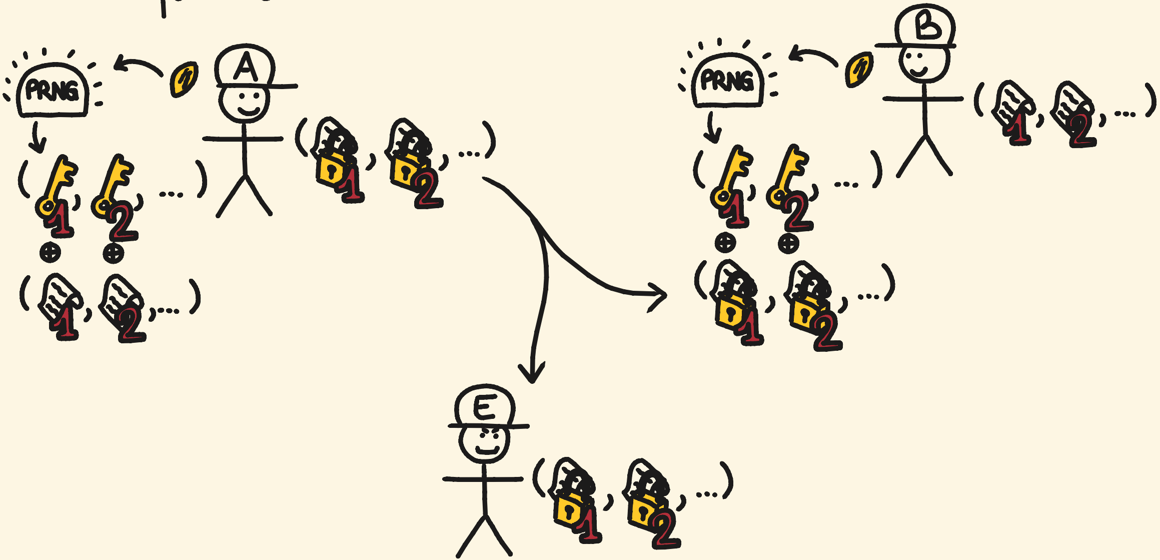


2. share a pseudorandom number generator



STREAM CIPHER PROTOCOL (2)

3. send a stream of encrypted messages through a public channel



STREAM CIPHER PROTOCOL

```
$\text{alice}^{\circ}(m)$ = do
    $\text{seedgen}() \rightsquigarrow ()$
    $\text{get}_A() \rightsquigarrow s$
    $\text{prng}(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$\text{alice}^{+\circ}(s, m)$ = do
    $\text{prng}(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$\text{alice}^{++} = \text{alice}^{+}$
```

```
$\text{bob}^{\circ}(m)$ = do
    $\text{get}_B() \rightsquigarrow s$
    $\text{prng}(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$\text{bob}^{+\circ}(s, m)$ = do
    $\text{prng}(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$\text{bob}^{++} = \text{bob}^{+}$
```

```
$\text{cipher}^{\circ}(m)$ = do
    $\text{alice}^{\circ}(m) \rightsquigarrow (s, m')$
    $\text{bob}^{\circ}(m') \rightsquigarrow (s', m'')$
    return $(s, s', m', m'')$

$\text{cipher}^{+\circ}(s_0, s_0', m)$ = do
    $\text{alice}^{+\circ}(s_0, m) \rightsquigarrow (s, m')$
    $\text{bob}^{+\circ}(s_0', m') \rightsquigarrow (s', m'')$
    return $(s, s', m', m'')$

$\text{cipher}^{++} = \text{cipher}^{+}$
```

COINDUCTION VS FEEDBACK

Streams are coinductive.

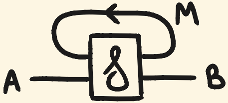
$$\mathrm{Stream}\ A = A \times \mathrm{Stream}\ A$$

Coalgebraic semantics.

$$\begin{tikzcd}
M \arrow[r] \arrow[d, dashed] & (M \times B)^A \arrow[d, dashed] \\
\Omega \arrow[r] & (\Omega \times B)^A
\end{tikzcd}$$

Feedback (pre)monoidal categories.

$$(M \mid f : M \otimes A \to M \otimes B)$$

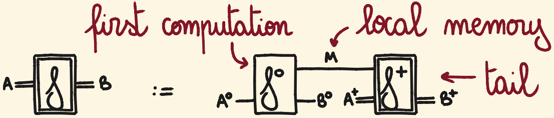


Sequential and parallel compositions.

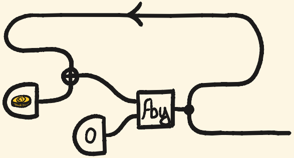
EFFECTFUL STREAMS: COINDUCTION AND FEEDBACK

Semantics of effectful reactive programs

- formal
- coinductive



- compositional



a random walk.

- effect agnostic

Stream : EffCat → EffCat

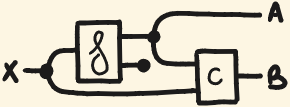
OUTLINE

- [Effectful categories]
- Effectful streams
- Causal processes
- Mealy machines, bisimulation and traces

STRING DIAGRAMS & DO-NOTATION

- Symmetric monoidal categories are theories of processes
- String diagrams and do-notation are convenient syntax

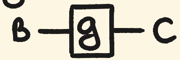
$$\nu_X ; ((f ; (\nu_A \otimes \varepsilon_B)) \otimes \mathbb{1}_X) ; (\mathbb{1}_A \otimes c)$$



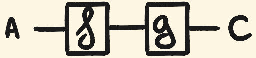
```
cond(x) = do
  f(x) → (a, b)
  c(a, x) → b'
  return (a, b')
```

STRING DIAGRAMS & DO-NOTATION

$f : A \longrightarrow B \,,\quad g : B \longrightarrow C$



- Composition. $f ; g : A \longrightarrow C$

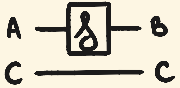


```
comp_fg(a) = do
  f(a) ~> b
  g(b) ~> c
  return(c)
```

- Identity. $id_A : A \longrightarrow A$

```
id_A(a) = do
  return(a)
```

- Whiskering. $w_C f : A \otimes C \longrightarrow B \otimes C$

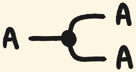


```
wh_fc(a,c) = do
  f(a) ~> b
  return(b,c)
```

- Symmetries. $\sigma_{A,B} : A \otimes B \rightarrow B \otimes A$

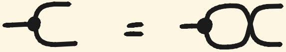
```
sym_AB(a,b) = do
  return(b,a)
```

COPY AND DISCARD

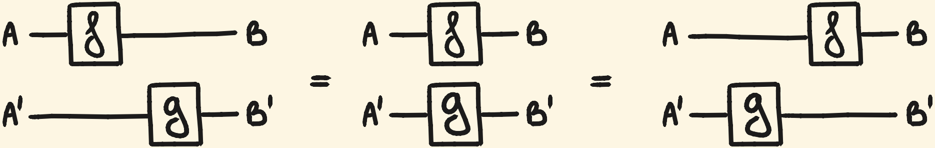


```
copy(a) = do
  return(a,a)
```

```
discard(a) = do
  return ()
```



THE INTERCHANGE LAW



```
par f g (a, a') = do
  f(a) → b
  g(a') → b'
  return (b, b')
```

=

```
par f g (a, a') = do
  g(a') → b'
  f(a) → b
  return (b, b')
```

⇝ holds in monoidal categories

COMPUTATIONS WITH EFFECTS

- Stochastic effects: generating the seed

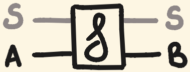
  $\mathcal{D} : \mathbf{Set} \to \mathbf{Set}$ distribution monad

  $\mathcal{D}(A) := \{ \sigma : A \to [0,1] \mid \operatorname{supp} \sigma \text{ is finite} \wedge \sum_{a \in A} \sigma(a) = 1 \}$

- Global state: sharing the seed

  $\mathrm{State}_S : \mathcal{C}^{op} \times \mathcal{C} \to \mathbf{Set}$ state promonad

  $\mathrm{State}_S(A, B) := \mathcal{C}(S \otimes A, S \otimes B)$



VALUES

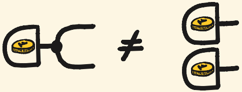
Values are both :

- deterministic

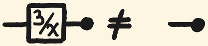
- total

ex $(3 \cdot -) : R \to \mathbb{R}$

NON-ex Flip : $1 \to \mathcal{D}(\{H,T\})$

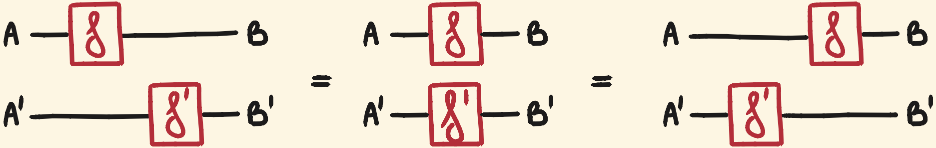


$(3/-) : R \to \mathbb{R}$



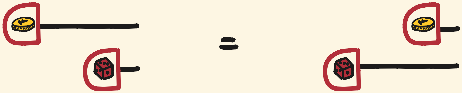
LOCAL COMPUTATIONS

Local computations interchange,



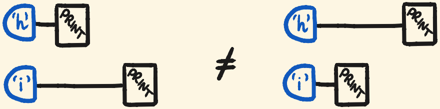
```
localF(a,a') = do                 localF(a,a') = do
  f(a) → b                            f'(a') → b'
  f'(a') → b'          =              f(a) → b
  return(b,b')                        return(b,b')
```

ex Stoch



EFFECTFUL COMPUTATIONS

Effectful computations may have global effects.



```
printHI() = do
  'h'() → c_1
  'i'() → c_2
  print(c_1) ⇝ ()
  print(c_2) ⇝ ()
  return ()
```

≠

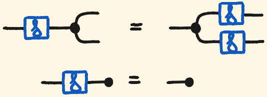
```
printIH() = do
  'h'() → c_1
  'i'() → c_2
  print(c_2) ⇝ ()
  print(c_1) ⇝ ()
  return ()
```

ex state promonads, IO monad

[Power and Robinson (1997)]

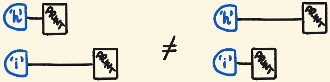
EFFECTFUL COPY-DISCARD CATEGORIES

Values can be copied and discarded (cartesian)

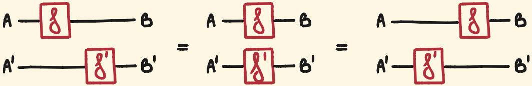


Effectful computations may have global effects (premonoidal)

$$\begin{tikzcd}\mathcal{V} \arrow[r] & \mathcal{L} \arrow[r] & \mathcal{C}\end{tikzcd}$$



Local computations interchange (monoidal)



ex (Set, Stoch, State_S)

(cart($\mathcal{C}$), $\mathcal{Z}$($\mathcal{C}$), $\mathcal{C}$) for a cd-premonoidal $\mathcal{C}$

[Jeffrey (1997), cf. Levy (2022), Power and Thielecke (1997)]

OUTLINE

- Effectful categories
- [Effectful streams]
- Causal processes
- Mealy machines, bisimulation and traces

STREAMS ARE COINDUCTIVE

A stream of elements of A is

- an element $a^0 \in A$
- a stream a^+ of elements of A

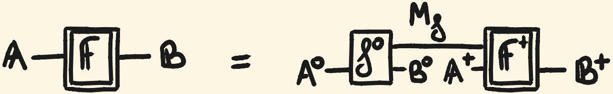
⇝ the set of streams is the final coalgebra of the functor

$$A \times (-) : \mathcal{S}et \to \mathcal{S}et$$

EFFECTFUL STREAMS

An *effectful stream* $F : A \to B$ on $(\mathcal{V}, \mathcal{L}, \mathcal{C})$ is

- a memory $M_f \in \mathcal{L}$
- a first action $f^\circ : A^\circ \to M_f \otimes B^\circ$ in $\mathcal{C}$
- the rest of the action $F^+ : M_f \cdot A^+ \to B^+$



quotiented by sliding

$$\begin{cases} f^\circ ; (r \otimes 1) = g^\circ \\ F^+ = r \cdot g^+ \end{cases}$$

for $r : M_f \to M_g$ in $\mathcal{L}$

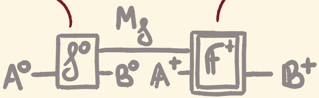
EFFECTFUL STREAMS

The profunctor Stream : $\mathcal{C}^{\mathbb{N}^{op}} \times \mathcal{C}^{\mathbb{N}} \longrightarrow \mathbf{Set}$ is the final coalgebra of the functor

$$F : [\mathcal{C}^{\mathbb{N}^{op}} \times \mathcal{C}^{\mathbb{N}}, \mathbf{Set}] \longrightarrow [\mathcal{C}^{\mathbb{N}^{op}} \times \mathcal{C}^{\mathbb{N}}, \mathbf{Set}]$$

$$F(Q)(A,B) := \int^{M \in \mathcal{C}} \mathcal{C}(A^{\circ}, M \otimes B^{\circ}) \times Q(M \cdot A^{+}, B^{+})$$

quotient by sliding on the memory



POLÝA URN

Consider (Set, Stoch, Stoch).

$\mathrm{pólya} : \mathbb{I} \to \mathbb{B}$

$\mapsto \mathbb{I}^{\circ} = I, \quad \mathbb{I}^{+} = \mathbb{I}$

$\mathbb{B}^{\circ} = \mathrm{Bool}, \quad \mathbb{B}^{+} = \mathbb{B}$

$$\mathrm{pólya}^{\circ}() = \mathrm{do}$$

$$\mathrm{Be}\left(\frac{b_0}{w_0 + b_0}\right) \to x$$

$$\mathrm{return}\,(\underbrace{w_0 + 1 - x,\ b_0 + x}_{\text{memory}:\ \mathbb{N} \times \mathbb{N}},\ x)$$

$\mapsto$ first action

$1 \to \mathbb{N} \times \mathbb{N} \times \mathrm{Bool}$

$$\mathrm{pólya}^{+\circ}(\overbrace{w, b}) = \mathrm{do}$$

$$\mathrm{Be}\left(\frac{b}{w + b}\right) \to x$$

$$\mathrm{return}\,(w + 1 - x,\ b + x,\ x)$$

$$\mathrm{pólya}^{++} = \mathrm{pólya}^{+}$$

$\mapsto$ tail, defined coinductively

$(\mathbb{N} \times \mathbb{N}) \cdot \mathbb{I} \to \mathbb{B}$

SEMANTICS OF STREAM CIPHER

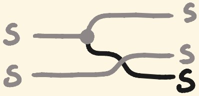
Consider (Set, Stoch, StateStoch),
where $\mathrm{StateStoch}(A,B) := \mathrm{Stoch}(S \otimes S \otimes A, S \otimes S \otimes B)$.

Fix $S = \mathbb{N}$, $C = \mathrm{Bool}^*$.

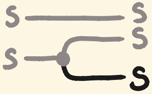
$\oplus : C \otimes C \to C$ ⇝ bitwise XOR

$\mathrm{prng} : S \to S \otimes C$ ⇝ pseudorandom number generator

$\mathrm{get}_A : I \rightsquigarrow S$



$\mathrm{get}_B : I \rightsquigarrow S$



$\mathrm{seedgen} : I \rightsquigarrow I$

STREAM CIPHER PROTOCOL

```
$alice^{\circ}(m)$ = do
    $seedgen() \rightsquigarrow ()$
    $get_A() \rightsquigarrow s$
    $prng(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$alice^{+\circ}(s, m)$ = do
    $prng(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$alice^{++} = alice^{+}$
```

```
$bob^{\circ}(m)$ = do
    $get_B() \rightsquigarrow s$
    $prng(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$bob^{+\circ}(s, m)$ = do
    $prng(s) \rightarrow (k, s')$
    return $(s', k \oplus m)$

$bob^{++} = bob^{+}$
```

```
$cipher^{\circ}(m)$ = do
    $alice^{\circ}(m) \rightsquigarrow (s, m')$
    $bob^{\circ}(m') \rightsquigarrow (s', m'')$
    return $(s, s', m', m'')$

$cipher^{+\circ}(s_0, s'_0, m)$ = do
    $alice^{+\circ}(s_0, m) \rightsquigarrow (s, m')$
    $bob^{+\circ}(s'_0, m') \rightsquigarrow (s', m'')$
    return $(s, s', m', m'')$

$cipher^{++} = cipher^{+}$
```

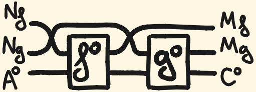
COMPOSITIONAL STRUCTURE OF STREAMS

THEOREM

Effectful streams form an effectful category Stream.

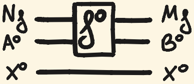
- composition and monoidal actions are defined coinductively: for $f : N_f \cdot A \to B$ and $g : N_g \cdot B \to C$,

$(f ;_N g)^\circ :=$



$(f ;_N g)^+ := f^+ ;_M g^+$

$(X \otimes_N f)^\circ :=$



$(X \otimes_N f)^+ := X^+ \otimes_M f^+$

FEEDBACK ON EFFECTFUL STREAMS

$\partial : \mathrm{Stream} \to \mathrm{Stream}$ $\qquad$ $\partial(A) := (I, A^{0}, A^{1}, \dots)$

THEOREM

Stream has ∂-feedback.

- feedback is defined coinductively

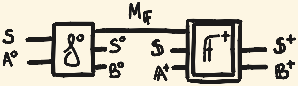
$$\mathbb{F} : (S \cdot \partial \mathbb{S}) \otimes A \to \mathbb{S} \otimes \mathbb{B}$$

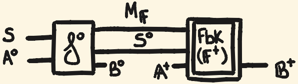
$$\mathrm{Fbk}_{\mathbb{S}}\,\mathbb{F} : S \cdot A \to \mathbb{B}$$

$$M(\mathrm{Fbk}^{S}_{\mathbb{S}}\,\mathbb{F}) := M(\mathbb{F}) \otimes \mathbb{S}^{\circ}$$

$$(\mathrm{Fbk}^{S}_{\mathbb{S}}\,\mathbb{F})^{\circ} := f^{\circ}$$

$$(\mathrm{Fbk}^{S}_{\mathbb{S}}\,\mathbb{F})^{+} := \mathrm{Fbk}^{S^{\circ}}_{\mathbb{S}^{+}}(\mathbb{F}^{+})$$





OUTLINE

- Effectful categories
- Effectful streams
- [Causal processes]
- Mealy machines, bisimulation and traces

STREAM COMPUTATIONS

- Sliding equivalence might be difficult to handle
- Causal stream functions are old :

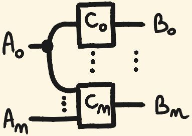
  [Raney (1958)] shows that they are
  the executions of deterministic Mealy machines

⇝ is there a similar explicit form for effectful streams ?

CAUSAL STREAM FUNCTIONS

Stream computations $(c_n)_{n\in\mathbb{N}} : A \to B$ in a cartesian category are families $c_n : A_0 \times \cdots \times A_n \longrightarrow B_n$.

⇝ $c_n(a_0, \dots, a_n) \in B_n$ is the output at time n



reconstructs the outputs until time n

[Sprunger & Katsumata (2019), Uustalu & Vene (2008)]

STOCHASTIC PROCESSES

Stochastic stream computations $(p_n)_{n\in\mathbb{N}} : \mathbb{A} \to \mathbb{B}$ are families $p_n : A_0 \times \dots \times A_n \longrightarrow \mathcal{D}(B_0 \times \dots \times B_n)$ such that $p_n(a_0,\dots,a_n) = \sum_{a\in A_{n+1}} p_{n+1}(a_0,\dots,a_n,a)$.

⇝ $p_n(a_0,\dots,a_n) \in \mathcal{D}(B_0 \times \dots \times B_n)$ is the distribution of the outputs <u>until</u> time n

↳ the outputs may be correlated

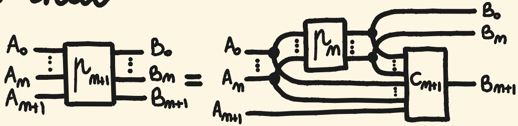
Is there a monoidal version of causal stream functions?

CAUSAL PROCESSES

A **causal process** $p : A \to B$ in a copy-discard category $\mathcal{C}$ is a family of morphisms

$$p_n : A_0 \otimes \cdots \otimes A_n \longrightarrow B_0 \otimes \cdots \otimes B_n$$

such that



for some $c_{n+1} : B_0 \otimes \cdots \otimes B_n \otimes A_0 \otimes \cdots \otimes A_n \otimes A_{n+1} \longrightarrow B_{n+1}$

⇝ p_n determines the input-output behaviour *until* time n

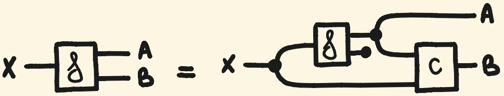
COMPOSING CAUSAL PROCESSES

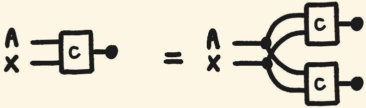
$\mathcal{C}$ copy-discard

CONDITIONALS WITH SHARP DOMAIN

[Cho & Jacobs (2017), Fritz (2020), EDL & Román (2023)]

For all $f : X \to A \otimes B$ there is $c : A \otimes X \to B$ st

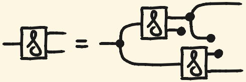
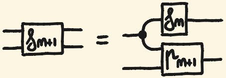
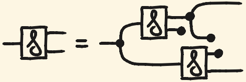
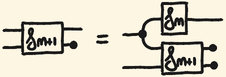
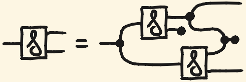
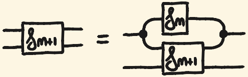
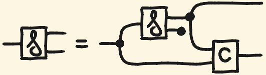
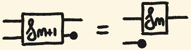
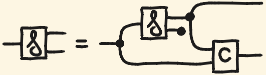




THEOREM

Causal processes form a monoidal category Proc when $\mathcal{C}$ has conditionals with sharp domain.

CAUSAL PROCESSES : EXAMPLES

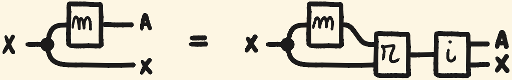
	CONDITIONALS	CAUSALITY CONDITION
Set		
Par		
Rel		
Stoch		
pStoch		

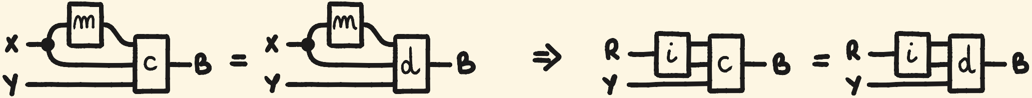
CAUSAL PROCESSES ARE STREAMS

$\mathcal{C}$ copy-discard

RANGES

For all $m : X \to A$ there are $\begin{cases} r : A \otimes X \to R & \text{deterministic} \\ i : R \to A \otimes X & \text{total} \end{cases}$





THEOREM

Consider ($\mathrm{fun}\,\mathcal{C}$, tot $\mathcal{C}$, $\mathcal{C}$).
If $\mathcal{C}$ has quasi-total conditionals and ranges,

$$\mathsf{Proc} \simeq \mathsf{Stream} .$$

OUTLINE

- Effectful categories
- Effectful streams
- Causal processes

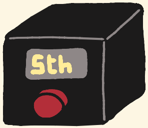
[• Mealy machines, bisimulation and traces]

MEALY MACHINES AND COALGEBRAS

Two faces of Mealy machines

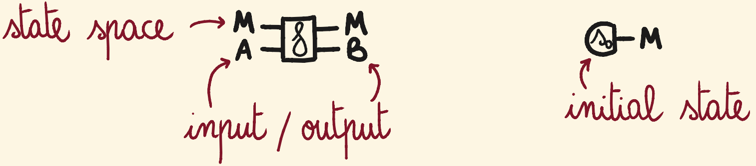
$$M \times A \to \mathcal{D}(M \times B) \qquad M \to \mathcal{D}(M \times B)^A$$

$$M \times A \to \mathcal{D}_c(M \times B) \qquad M \to \mathcal{D}_c(M \times B)^A$$



MEALY MACHINES

Systems are $\delta : M \otimes A \to M \otimes B$ with $s_0 : I \to M$



- native sequential and parallel compositions
- parametric in the underlying process theory
- premonoidal categories for global effects

⤳ what is their behaviour?
when are two of them equivalent?

[cf. Katis, Sabadini, Walters 1997]

COALGEBRAIC SEMANTICS

Systems are coalgebras $f : M \to F(M)$

input/output

$M \to (M \times B)^A$

non-determinism

$M \to \mathcal{P}(M \times B)$

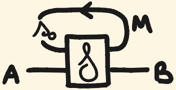
- bisimulation is equality in the final coalgebra

⇝ how do these compose?
how to change the underlying process theory?

EFFECTFUL TRACE SEMANTICS

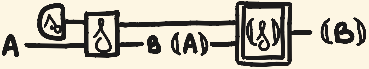
Fix a theory of programs.

effectful Mealy machines



↓

effectful streams



free construction
~ syntax

↓

coalgebraic construction
~ semantics

EFFECTFUL MEALY MACHINES

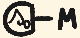
a **Mealy machine** $(f, M, s_0) : A \to B$ in $(\mathcal{V}, \mathcal{L}, \mathcal{C})$

is a morphism

$$f : M \otimes A \to M \otimes B$$

with an initial state

$$s_0 : I \to M$$



ex $(\mathbf{Set}, \mathbf{Rel}_{\mathrm{TOT}}, \mathbf{Rel})$

$$\begin{cases} f : M \times A \to \mathcal{P}(M \times B) \\ s_0 \subseteq M \end{cases}$$

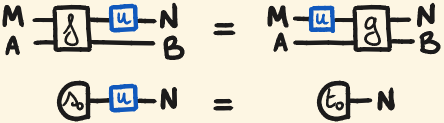
ex $(\mathbf{Set}, \mathbf{Stoch}, \mathbf{Stoch})$

$$\begin{cases} f : M \times A \to \mathcal{D}(M \times B) \\ s_0 \in \mathcal{D}(M) \end{cases}$$

[cf. Katis, Sabadini, Walters (1997); EDL, Gianola, Román, Sabadini, Sobociński (2022)]

MORPHISMS OF MEALY MACHINES

A <u>morphism</u> of Mealy machines $u : (f, M, s_0) \to (g, N, t_0)$ is a value morphism $u : M \to N$ in $\mathcal{V}$ such that



<u>ex</u> (Set, $\mathsf{Rel}_{\mathsf{TOT}}$, Rel)

$$\forall s \in M \ \forall a \in A \ \forall t \in N \ \forall b \in B$$

$$(t, b) \in g(u(s), a) \Leftrightarrow \exists s' \in M \ u(s') = t \wedge (s', b) \in f(s, a)$$

<u>ex</u> (Set, Stoch, Stoch)

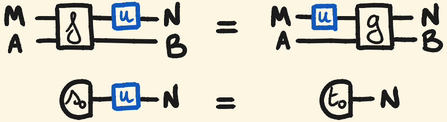
$$\forall s \in M \ \forall a \in A \ \forall t \in N \ \forall b \in B$$

$$g(t, b \mid u(s), a) = \sum_{s' : u(s') = t} f(s', b \mid s, a)$$

EFFECTFUL CATEGORY OF MEALY MACHINES

$\mathbf{Mealy}$ is an effectful category where

- objects are the objects of $\mathcal{C}$
- morphisms $(f, M, s_0) : A \to B$ are Mealy machines quotiented by value isomorphisms $u : M \xrightarrow{\cong} N$



- composition tensors the state spaces ⇝ local states

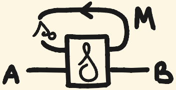
MEALY MACHINES ARE FREE

$\mathcal{S} := pt\mathcal{C}_{iso}$

THEOREM

Mealy is the free pointed-feedback category over $\mathcal{C}$.

$$\mathrm{Mealy}(A,B) = \int^{(s_0,M)\in pt\mathcal{C}_{iso}} \mathcal{C}(M\otimes A, M\otimes B)$$



[cf. Katis, Sabadini, Walters (1997); EDL, Gianola, Román, Sabadini, Sobociński (2022)]

COALGEBRAIC BISIMULATION

A bisimulation is a span of coalgebras.

$$\begin{tikzcd}
M \arrow[d, swap, "f"] & R \arrow[l, swap, "\pi_1"] \arrow[r, "\pi_2"] \arrow[d, "\alpha"] & N \arrow[d, "g"] \\
F(M) & F(R) \arrow[l, "F(\pi_1)"] \arrow[r, swap, "F(\pi_2)"] & F(N)
\end{tikzcd}$$

THEOREM [Rutten (2000)]

When $F : \mathbf{Set} \to \mathbf{Set}$ preserves weak pullbacks, bisimularity is an equivalence relation.

[Aczel & Mendler (1989), Rutten (2000)]

BISIMULATION

For two effectful Mealy machines $(f, M, s), (g, N, t) : A \to B$, a **bisimulation** is a sequence of spans of morphisms.

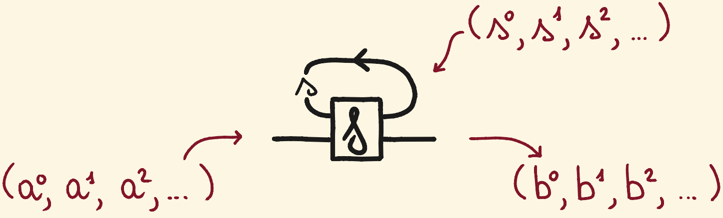
$$\begin{tikzcd}
 & (h_1, R_1, r_1) \arrow[dl, "u_1"'] \arrow[dr, "v_1"] & & \cdots \arrow[dl, "u_2"'] & (h_n, R_n, r_n) \arrow[dr, "v_n"] & \\
(f, M, s) & & (f_1, M_1, s_1) & & & (g, N, t)
\end{tikzcd}$$

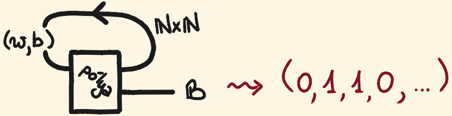
PROPOSITION

When $\mathcal{C} = Kl(T)$, for a commutative monad T preserving weak pullbacks, effectful bisimulation coincides with coalgebraic bisimulation.

ex Set, Par, Rel, Stoch, pStoch

EXECUTING MEALY MACHINES





⇝ what should the semantic universe be?

when do two Mealy machines have the same executions?

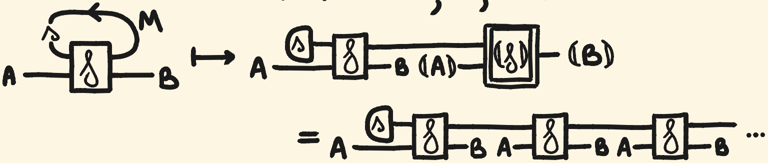
COMPOSITIONAL TRACE SEMANTICS

THEOREM

There is an effectful functor

$$Tr : Mealy \to Stream$$

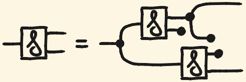
$$A \mapsto (\!|A|\!) = (A, A, \ldots)$$

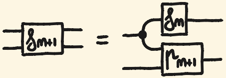


TRACES ARE EFFECTFUL TRACES

CONDITIONALS — CAUSALITY CONDITION — TRACE PREDICATE

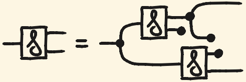
Set

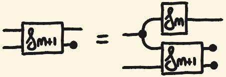




$s_0 = s \wedge \forall i \leq n \; (s_{i+1}, b_i) = f(s_i, a_i)$

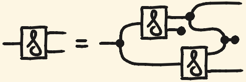
Par

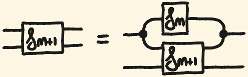




$s_0 = s \wedge \forall i \leq n \; (s_{i+1}, b_i) = f(s_i, a_i)$

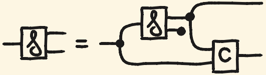
Rel

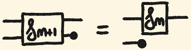




$\exists s_0, \dots, s_{n+1} \; s_0 \in s \wedge \forall i \leq n \; (s_{i+1}, b_i) \in f(s_i, a_i)$

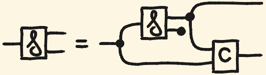
Stoch





$\sum_{s_0, \dots, s_{n+1}} s(s_0) \cdot \prod_{i \leq n} f(s_{i+1}, b_i \mid s_i, a_i)$

nStoch



$\sum_{s_0, \dots, s_{n+1}} s(s_0) \cdot \prod_{i \leq n} f(s_{i+1}, b_i \mid s_i, a_i)$

TRACE IS UNIVERSAL

THEOREM

The trace functor $\mathsf{Tr} : \mathsf{Mealy} \to \mathsf{Stream}$ is the unique feedback effectful functor determined by freeness of Mealy.

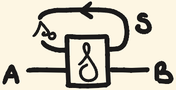
$$\begin{tikzcd}
\mathsf{FbkEffCat} \arrow[d, bend left] \\
\mathsf{EffCat} \arrow[u, bend left, "\mathsf{Mealy}"] \arrow[u, phantom, "\dashv"]
\end{tikzcd}
\quad \Rightarrow \quad
\exists!\ \mathsf{Tr} :
\begin{tikzcd}
\mathsf{Mealy}_{(\mathcal{V},\mathcal{L},\mathcal{C})} \arrow[rr] \arrow[dr] & {} \arrow[d, phantom, "\shortparallel"] & \mathsf{Stream}_{(\mathcal{V},\mathcal{L},\mathcal{C})} \\
& \mathsf{Mealy}_{(\mathcal{V},\mathcal{L},\mathcal{C})}/\mathsf{bisim} \arrow[ur] &
\end{tikzcd}$$

COROLLARY

Bisimilarity implies trace equivalence.

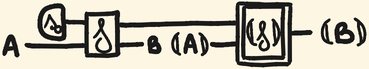
SUMMARY

effectful Mealy machines



↓ trace

effectful streams



≃ causal processes

free construction

~ syntax

↓ ∃!

coalgebraic construction

~ semantics

FUTURE WORK

- Adding choice, iteration and higher-order

- Coinduction up-to dinaturality
- Distributive law ?

- Linear temporal logic
- Behavioural metrics